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Let x, $x-ay$, $v-by$, $x-cy$ denote the four numbers required; then must $x^4+(x-ay)^4+(x-by)^4+(x-cy)^4=\square \ldots\ldots(1)$.

Expanding and arranging the terms according to the terms of x, we have $4x^4-4(a+b+c)x^3y+6(a^2+b^2+c^2)x^2y^2-4(a^3+b^3+c^3)xy^3+(a^4+b^4+c^4)y^4=\square \ldots\ldots(2)$; or, putting $a+b+c=m$, $a^2+b^2+c^2=n$, $a^3+b^3+c^3=p$,

$$a^4+b^4+c^4=q,\ 4x^4-4mx^3y+6nx^2y^2-4pxy^3+qy^4=\square \ldots\ldots(4).$$

Putting $(4)=[2x^2-mxy+\frac{1}{4}(6n-m^2)y^2]^2$, expanding and reducing, we find $\dfrac{x}{y}=\dfrac{(6n-m^2)^2-16q}{8m(6n-m^2)-64p}$,

$$=\frac{[6(a^2+b^2+c^2)-(r+b+c)^2]^2-16(a^4+b^4+c^4)}{8(a+b+c)[6(a^2+b^2+c^2)-(a+b+c)^2]-64(a^3+b^3+c^3)}.$$

If $a=5$, $b=2$, $c=1$, then $\dfrac{x}{y}=\dfrac{199}{-72}$, and we may take $x=199, y=-72$.

These values give the numbers 199, 271, 343, and 559.

$$\therefore\ 199^4+271^4+343^4+559^4=344162^2.$$

14. Proposed by **SYLVESTER ROBINS**, North Branch Depot, New Jersey.

Find initial terms in each of three infinite series of prime, integral, rational, scalene triangles, where 9 shall be the base, and the other two sides of every term shall have a constant difference.

Solution by **ARTEMAS MARTIN, LL D**, U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let $x+\frac{1}{2}d$ and $x-\frac{1}{2}d$ be "the other two sides;" then the area of the triangle is $\surd[(x+4\frac{1}{2})(4\frac{1}{2}-\frac{1}{2}d)(4\frac{1}{2}+\frac{1}{2}d)(x-4\frac{1}{2})]$, which must be rational, or $(20\frac{1}{4}-\frac{1}{4}d^2)(x^2-20\frac{1}{4})=\square \ldots\ldots(1)$.

Put $20\frac{1}{4}-\frac{1}{4}d^2=c^2e$, then (1) becomes $e(x^2-20\frac{1}{4})=\square \ldots\ldots(2)$.

Put $e(x^2-20\frac{1}{4})=\dfrac{p^2}{q^2}(x+4\frac{1}{2})^2$, then we get $x=\dfrac{4\frac{1}{2}(eq^2+p^2)}{eq^2-p^2}$;

$$\therefore\ x+\tfrac{1}{2}d=\frac{(4\frac{1}{2}+\frac{1}{2}d)eq^2+(4\frac{1}{2}-\frac{1}{2}d)p^2}{eq^2-p^2},\ x-\tfrac{1}{2}d=\frac{(4\frac{1}{2}-\frac{1}{2}d)eq^2+(4\frac{1}{2}+\frac{1}{2}d)p^2}{eq^2-p^2}.$$

Now eq^2-p^2 may have any value, positive or negative, that will exactly divide the numerators of the expressions for $x+\frac{1}{2}d$ and $x-\frac{1}{2}d$.

It is plain that d must be an odd number, and it must be less than 9, and therefore can not be greater than 7.

I.—Take $d=1$, then $e=5$, and $5q^2 \backsim p=1$ will satisfy this case. The least values of p and q are $p=2$, $q=1$, which give 40 and 41 for "the other two sides." Therefore the sides of the first triangle of the series having 9 for bases and a constant difference of unity between the other two sides are 9, 40, 41.

II.—When $d=3$, we have the numerator of $x+\frac{1}{2}d=6eq^2+3p^2=3(2eq^2+p^2)$, and the numerator of $x-\frac{1}{2}d=3eq^2+6p^2=3(eq^2+2p^2)$, both of which are always divisible by 3 whatever the values of p and q; hence there can not be "prime, integral, rational, triangles" having 9 for base and 3 for difference of the other two sides.

III.—When $d=5$, then $e=14$. In this case take $14q^2 \backsim p^2=2$; the least values of p and q are $p=4$, $q=1$, and we find the sides of the first triangle in this case to be 9, 60, 65.

IV.—When $d=7$, then $e=2$, and $2q^2 \backsim p^2=1$ will give the required results. Taking $p=1$, $q=1$, we find the sides of the first triangle to be 9, 10, 17.

II. Solution by the PROPOSER.

I. Let 9, $x-\frac{1}{2}$, $x+\frac{1}{2}$ be sides of a rational △.

Then $(x+4\frac{1}{2})(x-4\frac{1}{2})5\times4=\square$ of area. $5(x^2-20\frac{1}{4})=\square$.

Key $=\sqrt{5}=\frac{2}{1}, \frac{9}{4}, \frac{38}{17}, \frac{161}{72}, \frac{682}{305}, \frac{2889}{1292}$, &c.

$5\times4^2+1^2=81$,	I. △	9	40	41,
$5\times17^2+2^2=1449$,	II.	9	724	725,
$5\times72^2+9^2=26001$,	III.	9	13000	13001,
$5\times305^2+38^2=466569$,	IV.	9	233284	233285,
$5\times1292^2+161^2=8372241$,	V.	9	4186120	4186121, &c.

II. Let 9, $x-2\frac{1}{2}$, $x+2\frac{1}{2}$=sides of △. $(x+4\frac{1}{2})(x-4\frac{1}{2})7\times2=\square$.

$14(x^2-20\frac{1}{4})=\square$. Key $=\sqrt{14}=\frac{3}{1}, \frac{4}{1}, \frac{11}{3}, \frac{15}{4} \,..\, \frac{449}{120} \,..\, \frac{13455}{3596} \,..\, \frac{403201}{107760}$, &c.

$9\times15=135$,	I. △	9	65	70,
$9\times449=4041$,	II.	9	2018	2023,
$9\times13455=121095$,	III.	9	60545	60550,
$9\times403201=3628809$,	IV.	9	1814402	1814407,
$9\times12082575=108743175$,	V.	9	54371585	54371590, &c.

III. Let 9, $x-3\frac{1}{2}$ and $x+3\frac{1}{2}$=sides of △.

$(x+4\frac{1}{2})(x-4\frac{1}{2})\ 8\times1=\square$. $2(x^2-20\frac{1}{4})=\square$.

Key $=\sqrt{2}=\frac{1}{1}, \frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{99}{70}, \frac{239}{169}, \frac{577}{408}$, &c.

$5^2+2=27$,	I. △	9	10	17,
$12^2+3^2+0=153$,	II.	9	73	80,
$29^2+7^2+1^2=891$,	III.	9	442	449,
$70^2+17^2+2^2=5193$,	IV.	9	2593	2600,
$169^2+41^2+5^2=30267$,	V.	9	15130	15137,
$408^2+99^2+12^2=176409$,	VI.	9	88201	88208, &c.

P. S. BERG refers to the proposer's article on pp. 262-3 of August MONTHLY.

PROBLEMS.

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20 **Proposed by G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

Find two integral numbers, whose sum, difference, and difference of their squares shall each be a square, cube and fourth power.